

# Decision-Making Under Uncertainty

## Week 2 — Value, Part 1: Behavioral and Economic Foundations

---

Lecturers:

Prof. Kerstin Preuschoff

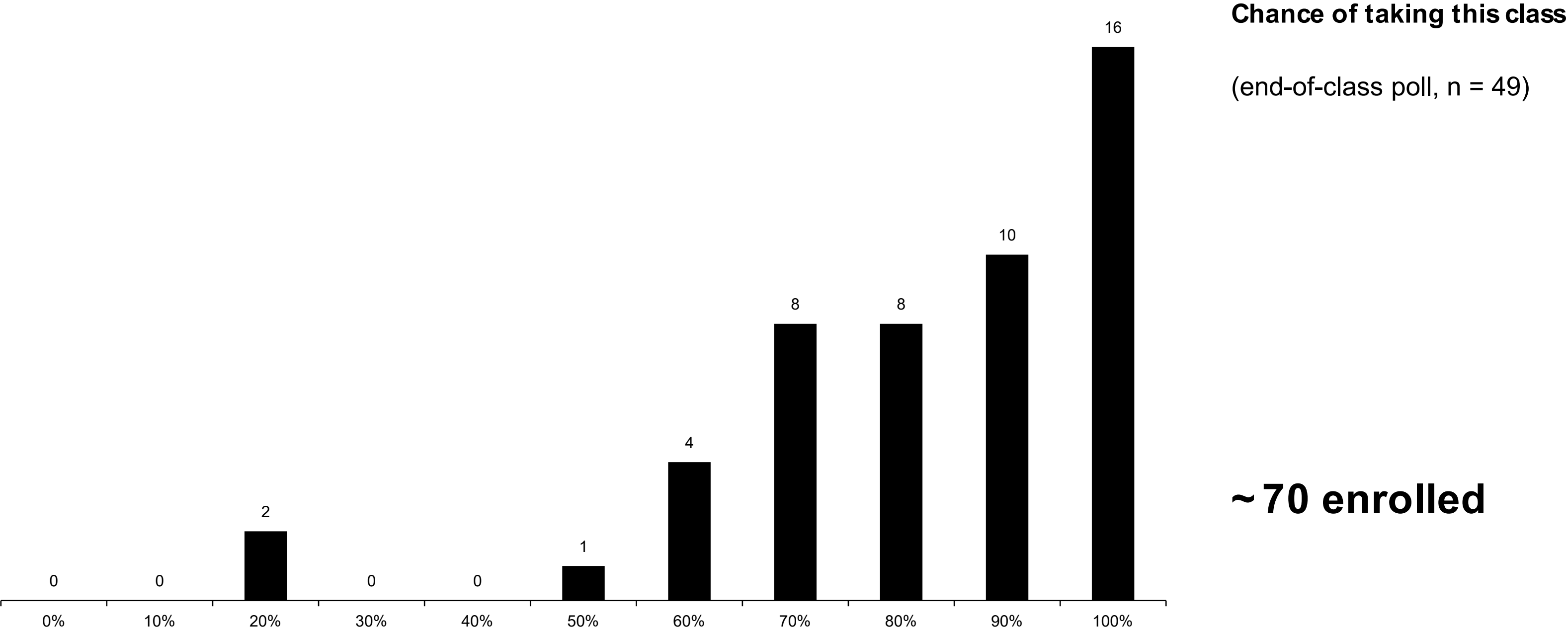
Dr. Marco Lehmann

TA: Jean-Paul Nohra

22.09.2026

# What I value, and what I can count

---



# What do you value?

---

In one word— no wrong answers, first thing that comes to mind.

 **Moodle -> Padlet — open text:** → post whatever you value, in your own words.

# What Do You Value?

Use simple terms. ex: family, integrity, nature...

 ag  
4 hours ago

family

 14

 Explorateur Toundra  
4 hours ago

health

 13

 Explorateur Toundra  
4 hours ago

time

 5

 om  
4 hours ago

love



 om  
4 hours ago

self-realization

 2

 Ospite Praterie  
4 hours ago

time



 om  
4 hours ago

self-fulfillment

 2

 Wolken Reiziger  
4 hours ago

safety



 Icefall Climber  
4 hours ago

fun



 AM  
4 hours ago

discipline

 3

 Mon  
4 hours ago

Ecology



 Courteous Anchovy  
4 hours ago

friendship, emotional stability,  
health, love,

 1

 Brilliant Beetle  
4 hours ago

Happiness

 1

 IMD  
4 hours ago

Football

 2

 Bâtitteur Pionnier  
4 hours ago

effort

 1

 Hoogland Klauteraar  
4 hours ago

friends

 2

 cc  
4 hours ago

experience

 2

 Canvas Keeper  
4 hours ago

family



 Gnetteur Bravoure  
4 hours ago

Work



 Discovery Mapper  
4 hours ago

Freedom

 4

# Same three lenses, aimed at a decision

|                        | Economics /Finance                                                               | Psychology                                                                                                      | Neuroscience                                                              |
|------------------------|----------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------|
| Core question          | Given constraints and information, what choice maximizes expected value/utility? | What cognitive and emotional processes produce this choice, and where does it depart from rational calculation? | What neural computations and circuits implement the choice?               |
| What counts as “value” | Money, wealth, expected returns — one comparable currency.                       | Subjective well-being, satisfaction, felt importance — often domain-specific.                                   | Reward and prediction-error signals — a candidate common neural currency. |
| Typical methods        | Formal modeling, revealed preference, econometrics, natural experiments.         | Controlled experiments, self-report, behavioral observation.                                                    | fMRI, EEG, single-unit recording, lesion studies.                         |

*No field has ground truth.*

# Similar words, different meanings and histories

| Term            | Economics                               | Psychology                    | Neuroscience                   |
|-----------------|-----------------------------------------|-------------------------------|--------------------------------|
| Value / Utility | Bernoulli 1738 → Pareto 1906 → VNM 1944 | Fechner 1860 → Atkinson 1957  | Reward prediction error (Wk 4) |
| Gain            | Reference-dependence (Wks 7–8)          | Not this era's term (Wks 7–8) | —                              |
| Reward          | Not econ's vocabulary                   | Thorndike 1898 → Skinner 1938 | Dopamine / RPE (Wk 4)          |
| Wealth          | The default currency                    | Not used as a unit            | —                              |
| Need / Drive    | —                                       | Maslow 1943                   | —                              |
| Expectancy      | —                                       | Tolman 1932 → Atkinson 1957   | —                              |

# Classical Economic Theories

---

"Father" of economics — and it's 'self-love,' not money

---

*"It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard to their own self-interest. We address ourselves not to their humanity but to their self-love, and never talk to them of our own necessities, but of their advantages."*

- **Adam Smith, The Wealth of Nations (1776), Book I, Chapter II**

# From Blaise Pascal to Intuitive/Cardinal Utility (1600s-1870s)

# Developing economic theories

---

- 1600's Blaise Pascal introduces probability theory by solving the following problem:
  - “Two people, A and B, agree to play a series of fair games until one person has won six games. They each have wagered the same amount of money, the intention being that the winner will be awarded the entire pot. But suppose, for whatever reason, the series is prematurely terminated, at which point A has won five games and B three. How should the stakes be divided?”
- This led to the development of probability theory and the concept of expected value.

*Discussion: Knowing that probability and expected value had not yet been “invented” how would A and B have argued at the time? What solution would either one of them have proposed and why?*

# Computing expected value

---

$$EV(X) = \sum p(x) \cdot x$$

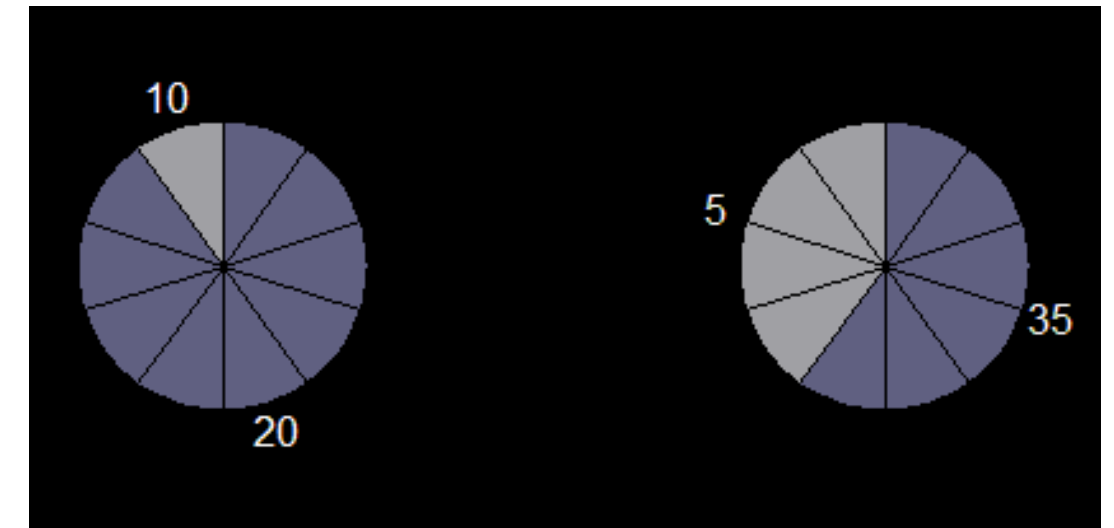
A probability-weighted average of possible outcomes.

**Example 1:** 10% chance of CHF 10, 90% chance of CHF 20

$$\rightarrow EV = 0.1 \times 10 + 0.9 \times 20 = \text{CHF } 19$$

**Example 2:** 40% chance of CHF 5, 60% chance of CHF 35

$$\rightarrow EV = 0.4 \times 5 + 0.6 \times 35 = \text{CHF } 23$$



# What I value, and what I can count

---

| Chance | # students | Probability | Contribution |
|--------|------------|-------------|--------------|
| 0%     | 0          | 0.0         | 0.0          |
| 10%    | 0          | 0.1         | 0.0          |
| 20%    | 2          | 0.2         | 0.4          |
| 30%    | 0          | 0.3         | 0.0          |
| 40%    | 0          | 0.4         | 0.0          |
| 50%    | 1          | 0.5         | 0.5          |
| 60%    | 4          | 0.6         | 2.4          |
| 70%    | 8          | 0.7         | 5.6          |
| 80%    | 8          | 0.8         | 6.4          |
| 90%    | 10         | 0.9         | 9.0          |
| 100%   | 16         | 1.0         | 16.0         |

Total (n = 49):

**40.3 expected  
attendees**

*~70 students enrolled.*

*Today:  $p$  is your likelihood score,  $x$  is one more student in the room.*

# Falsifying expected value: The St. Petersburg Paradox

---

- First stated in 1713 by Nicolas Bernoulli, and solved in 1738 by his brother Daniel Bernoulli
- You are offered to buy a lottery ticket. The prizes in this lottery are determined by tossing a fair coin
- You receive:
  - CHF 2 if “heads” come up on the very first toss
  - CHF 4 if “tails” come up on the first toss and “heads” come up on the second toss
  - CHF 8 if “tails” come up on the first and the second toss and “heads” come up on the third toss
  - ... CHF  $2^n$  if “heads” come up for the first time on the  $n$ th toss of the coin
- What is a maximum price that will you pay for this lottery ticket?

How would you compute the expected value (according to Pascal)?

---

## Applying expected value theory to the problem

---

- Typically, people are willing to pay less than CHF 10 for the St. Petersburg lottery
- This lottery offers an outcome CHF  $2^n$  with probability  $1/(2^n)$ ,  $n \in \mathbb{N}$
- The expected value of the St. P. lottery  $EV(L) = 2 \times 1/2 + 4 \times 1/4 + \dots + 2^n \times 1/(2^n) + \dots = 1 + 1 + \dots + 1 = +\infty$
- **Someone who maximises expected value should offer all his wealth for the right to play in the St. Petersburg lottery**

## Solving the problem (Bernoulli)

---

- Observe that the concept of expected value is not a descriptive decision theory
  - it fails to describe the observed behaviour as people do not bet all their wealth on the St. Petersburg lottery
- It is also not a normative decision theory: Should people ever bet all their wealth?
- A solution was proposed by Swiss mathematician Daniel Bernoulli in an article published in 1738 in the proceedings of St. Petersburg Academy of Sciences.

# The dilemma with Pascal's expected value

---

- $EV(X) = \sum (p(x) * x)$ 
  - Option 1: CHF 50 for sure
  - Option 2: toss a coin, CHF 105 if heads, CHF 0 otherwise
- EV theory says that you should prefer option 2, many people prefer option 1

|                | Option 1 | Option 2 |
|----------------|----------|----------|
| Value          | 50       | 105      |
| Probability    | 1        | 1/2      |
| Expected value | 50       | 52.5     |

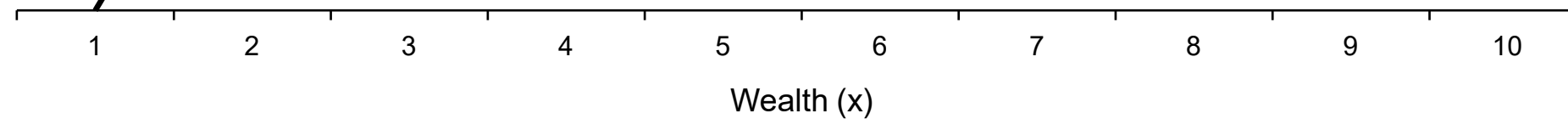
# Diminishing marginal utility

---

$$u(x) = \log(x)$$

The same increase in wealth adds less utility the more you already have.

*“The second million doesn't feel like the first.”*



*Cf. Self-study. Weber-Fechner law*

---

*“There is no doubt that a gain of one thousand ducats is  
is more significant to a pauper than to a rich man, though  
though both gain the same amount.”*

— Daniel Bernoulli, 1738

# What kind of utility is this?

---

## Bernoulli (1738)

---

|                           |                                         |
|---------------------------|-----------------------------------------|
| Type                      | Cardinal                                |
| Meaning                   | Psychological — a real mental sensation |
| Measurable?               | No — introspective only                 |
| Comparable across people? | Claimed yes                             |

*This table gains a column at §3 (Pareto) and again at §5 (von Neumann–Morgenstern) — full comparison recapped at close.*

# Why is water cheap and diamonds expensive?

---

*"The things which have the greatest value in use have frequently little or no value in exchange; and, on the contrary, those which have the greatest value in exchange have frequently little or no value in use. Nothing is more useful than water: but it will purchase scarce any thing... A diamond, on the contrary, has scarce any value in use; but a very great quantity of other goods may frequently be had in exchange for it.*

**Adam Smith, The Wealth of Nations (1776)**

|          | Total utility | M arginal utility | Price |
|----------|---------------|-------------------|-------|
| Water    | High          | Low               | Low   |
| Diamonds | Low           | High              | High  |

*Price tracks the value of the last unit consumed — not the value of the whole category.*

➤ **Solved independently by 3 economists in the 1870s -> Jevons, Menger, Walras**

# Marginal Utility Theory

---

- Marginal utility explains value
- Value = marginal utility (utility of the last unit consumed), not total utility
- Diminishing marginal utility - Each additional unit provides less satisfaction

## **Still Cardinal and Psychological**

- Despite the advance, marginal utility theory still treated utility as:
  - Cardinal (measurable on an absolute scale)
  - Psychological (representing actual mental satisfaction)

# The ordinal revolution: Vilfredo Pareto (1906)

---

## Key Insight:

- Economics doesn't need cardinal utility!
- Only need to know **preference rankings**:  $A > B > C$
- Don't need to know "how much" someone prefers A to B

## Vilfredo Pareto's Argument:

- "All we observe is choice, not inner satisfaction"
- Economics should be based on observable behavior
- This is **ordinal utility** - only the ranking matters



?



# The ordinal revolution: Vilfredo Pareto (1906)

---

## Advantages:

- No need to measure "happiness"
- No interpersonal comparisons needed
- More scientifically respectable
- $utility(A)=10$ ,  $utility(B)=9$ ,  $utility(C)=8$  contains same information as  $utility(A)=1000$ ,  $utility(B)=50$ ,  $utility(C)=1$

## Transformation Property

- If  $u(x)$  represents preferences, then  $f(u(x))$  also represents the same preferences for any strictly increasing function  $f$ .



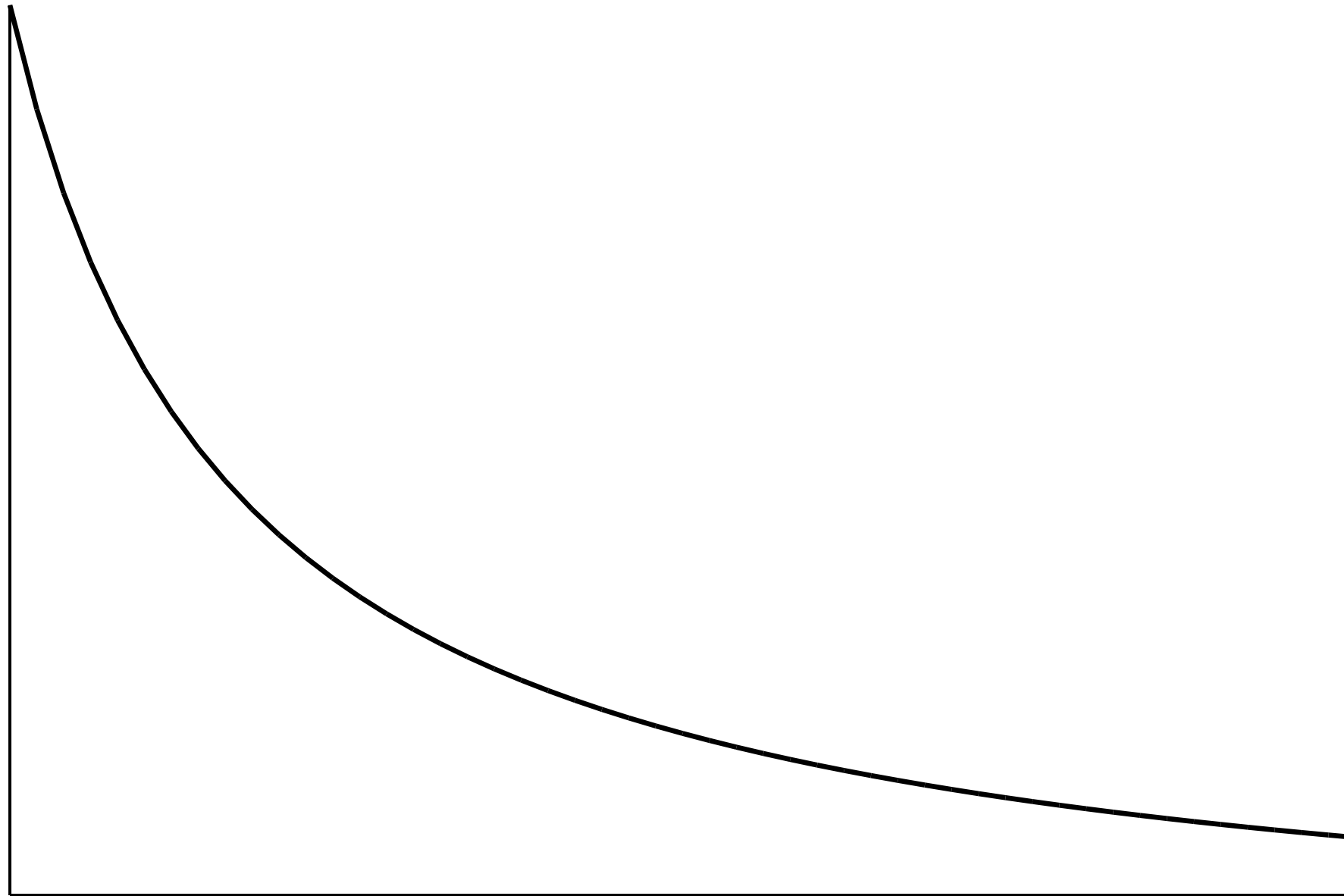
?



# Ranking without measuring (illustrative)

---

↑ Perceived durability / values-fit



Price premium tolerated →

**Every point on this curve is equally preferred.**

You don't need a number for “ how someone values sustainability to describe their choice this way — only that they treat these bundles as interchangeable.

# What kind of utility is this? (updated)

---

|                           | Bernoulli (1738)                        | Pareto (1906)                                       |
|---------------------------|-----------------------------------------|-----------------------------------------------------|
| Type                      | Cardinal                                | Ordinal                                             |
| Meaning                   | Psychological — a real mental sensation | Mathematical convenience, not psychological reality |
| Measurable?               | No — introspective only                 | Yes — revealed through choices                      |
| Comparable across people? | Claimed yes                             | No                                                  |

*Gains its final column at §5 (von Neumann–Morgenstern) — recapped in full at close.*

# The axiomatic revolution & Generalised axiomatisation

# The Crisis of Utility Theory

---

## Challenges

- How to handle choices under uncertainty? Ordinal utility works for certain outcomes, but what about risky choices?
- How to make utility theory rigorous? Need mathematical foundations
- How to avoid "mentalist" concepts? Unobservable mental states are problematic

## Solutions

- Theory of Value (Hicks & Allen, 1934) -> finishes Pareto
- Revealed Preference Theory (Samuelson, 1938) -> removes theory itself
- Expected Utility Theory (von Neumann and Morgenstern, 1944) -> handles risk

# The Crisis of Utility Theory. Solutions

---

## Solutions

- Theory of Value (Hicks & Allen, 1934)
  - formalized Pareto insights which were purely conceptual; in practice economists were still using cardinal marginal utility
  - purify the *numbers* (cardinal  $\rightarrow$  ordinal)
  - Moved from marginal utility to marginal rate of substitution
- Revealed Preference Theory (Samuelson, 1938)
  - purify the *existence claim* (no assumed preferences)
  - Just observe choice, everything can be extracted from this
- Expected Utility Theory (von Neumann and Morgenstern, 1944)
  - handles risk (NOT uncertainty in general)

# Samuelson

---

- Start from simple, “primitive”, assumptions on preferences.
- Use observed choices, NOT UTILITY
- Derive preference ordering from choice consistency.
- Construct mathematical structure of choice.

## Key Achievement:

- No mention of utility, satisfaction, or mental states
- Pure behaviorism
- If choices satisfy WARP, we can say they're "consistent with utility maximization"
- "As if" methodology - acts as if maximizing some utility function
- Normative models that focus on idealised choices and efficient allocation of resources.
- (as opposed to necessarily seeking to describe how people choose, as psychologists do, and how markets work).

# Describing choice

---



?



# Representing choice

---



10

>



>

9

# The Axiomatic Approach to Modeling

---

Goal: Formalize models through representation theorems

i.e., find mathematical structures that “represent” the choices we observe

e.g.

$$U(x) > U(y) \iff x \succ y$$

numbers

preferences

articulate a set of necessary & sufficient conditions for representation

e.g., “transitivity,”

$$a \succ b, b \succ c \Rightarrow a \succ c$$

# Representing choice

---



X

?



?

9

# Representing choice

---



8

∧



9

∧

# Inferring choice

---



>



<



8

>

<



10

# Observing and inferring from choice

---



>



<



>



$\text{utility}(\text{bananas}) = 8 < \text{utility}(\text{apples}) = 10$

# Utility functions

---

- used to represent individual's choice
- any function that results in the correct ordering works  
e.g.,  $\text{utility}(\text{banana})=13$ ,  $\text{utility}(\text{orange})=70$ ,  $\text{utility}(\text{apple}) = 1053$
- only represent choice and have no independent explanatory power  
i.e., I cannot extract the value of an option from its utility
- in their “purest” form they are merely a tool to capture choice
- different from *expected utility* discussed above

# Consistency axioms

---

- SARP\* - an individual's choices are transitive
- WARP\* - if an individual chooses an orange over an apple when both are available in situation 1, then the individual should not choose the apple over the orange in another situation where again both are available
- questions about the neural and psychological processes behind choices are irrelevant as long as human decision making obeys such simple consistency axioms

\*strong/weak

axiom of revealed preference  
Houthakker, 1950

# The beauty and the irony of revealed preference

---

- The beauty:  
based on very few, very simple axioms, economists can make predictions about surprisingly complex choice patterns  
predicts what new choices could possibly follow from an observed set of previous choices
- The irony:  
it represented the “nail in the coffin” for psychological influences on economics

Von Neumann & Morgenstern (1944)  
"Theory of Games and Economic Behavior"

# The problem

---

- Ordinal utility works for choices under certainty
- But under **uncertainty**, we need more structure
- Example: If I prefer A to B to C, how do I choose between:
  - Lottery 1: 50% chance of A, 50% chance of C
  - Lottery 2: 100% chance of B?

# Which would you choose?

---

## Pair 1

A: CHF 3,000 for sure

B: 50% chance of CHF 7,000, 50% chance of CHF 0

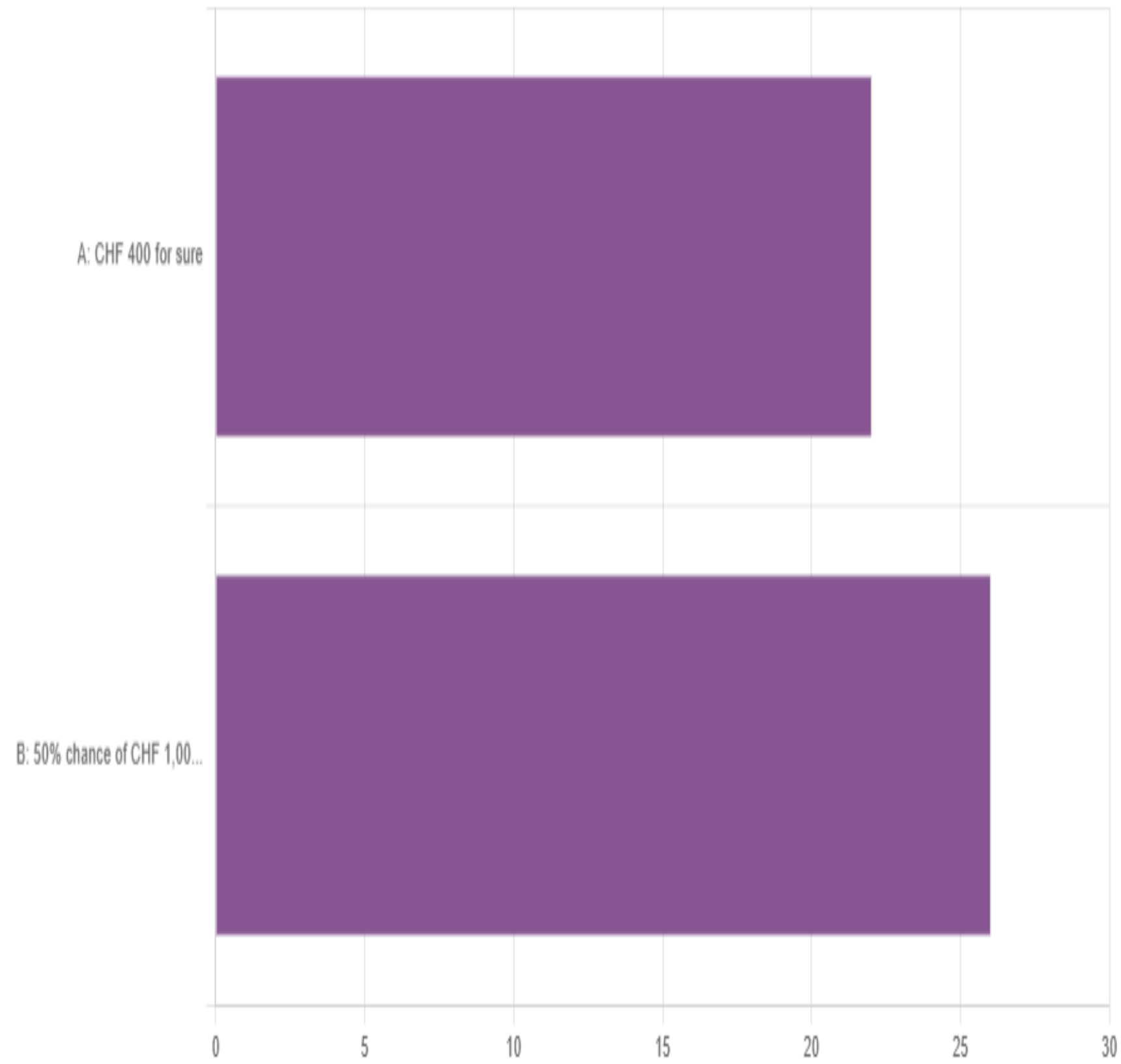
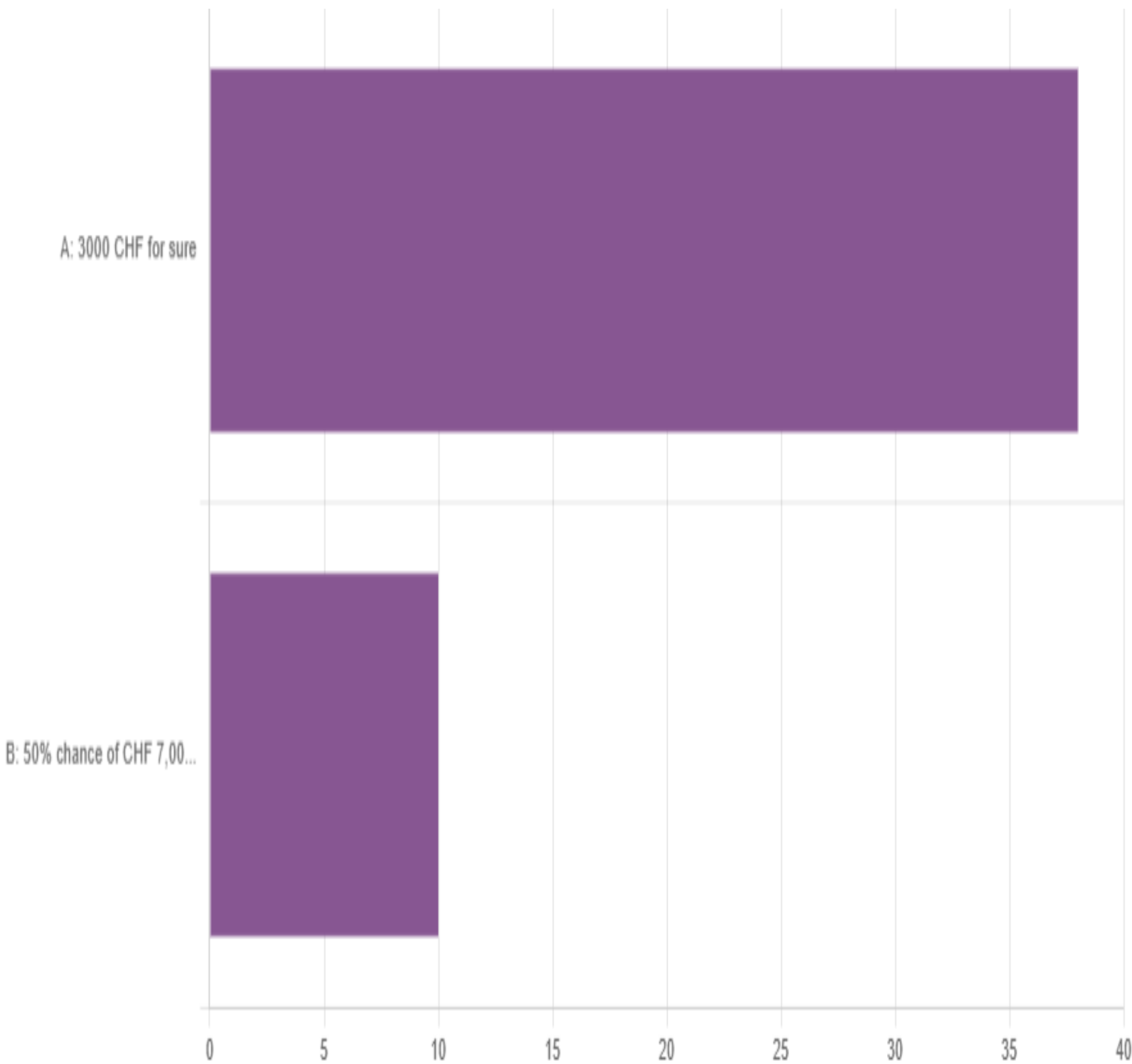
## Pair 2

A: CHF 400 for sure

B: 50% chance of CHF 1,000, 50% chance of CHF 0

 Moodle — for each pair: → A or B?

# Which would you choose?



# The solution: The axiomatic approach

---

They proposed **axioms** for rational choice under uncertainty:

- 1. Completeness:** Can compare any two lotteries
  - 2. Transitivity:** If  $A \succcurlyeq B$  and  $B \succcurlyeq C$ , then  $A \succcurlyeq C$
  - 3. Continuity:** If  $A \succ B \succ C$ , there exists probability  $p$  such that  $B \sim pA + (1-p)C$
  - 4. Independence:** If  $A \succ B$ , then  $pA + (1-p)C \succ pB + (1-p)C$  for any lottery  $C$  and probability  $p$  (*note: see Allais paradox, Week 7*)
- Also showed how to recover utility from choices

# The representation theorem

---

If preferences satisfy completeness, transitivity, continuity, and independence, exists a utility function  $u(\cdot)$  such that:

$$U(L) = \sum p(x) \cdot u(x)$$

and the decision-maker acts *as if* maximizing expected utility.

*$u(\cdot)$  is unique only up to a positive affine transformation: if  $u(x)$  represents these preferences, so does  $a \cdot u(x) + b$  for any  $a > 0$ .*

# Recovering utility from choices

---

| Observed choice                                                      | What it reveals               |
|----------------------------------------------------------------------|-------------------------------|
| Prefers CHF 3,000 for sure over 50/50 of CHF 7,000 or CHF 0          | $u(3000) > 0.5 \cdot u(7000)$ |
| Indifferent between CHF 400 for sure and 50/50 of CHF 1,000 or CHF 0 | $u(400) = 0.5 \cdot u(1000)$  |

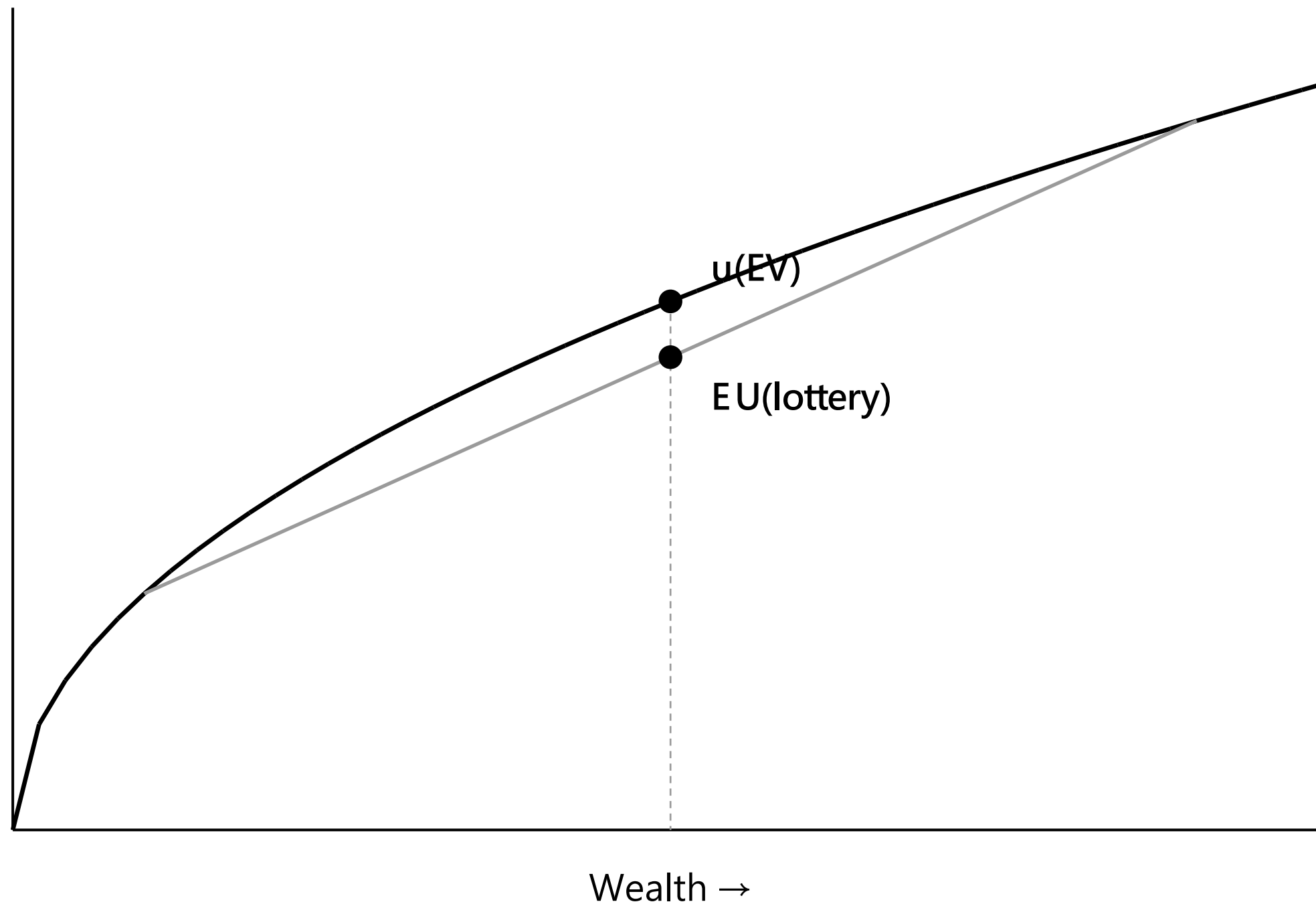
**Normalize  $u(0) = 0$ ,  $u(7000) = 1$ :**

→  $u(3000) > 0.5$  — that inequality *is* risk aversion, numerically.

→  $u(400) = 0.5$

*NOTE: two separate rulers (one per gamble), not one scale.*

# Concavity is risk aversion



A concave  $u(\cdot)$  means:

$$EU(\text{lottery}) < u(EV \text{ of lottery})$$

The room's own preference for the certain amount over the higher-EV gamble *is* this curve, visually.

# What kind of utility is this? (final)

|                           | Bernoulli (1738)                        | Pareto (1906)                                       | Von Neumann–Morgenstern (1944)                   |
|---------------------------|-----------------------------------------|-----------------------------------------------------|--------------------------------------------------|
| Type                      | Cardinal                                | Ordinal                                             | Cardinal — again, different meaning              |
| Meaning                   | Psychological — a real mental sensation | Mathematical convenience, not psychological reality | Derived from consistent preferences, not assumed |
| Measurable?               | No — introspective only                 | Yes — revealed through choices                      | Yes — via lottery choices                        |
| Comparable across people? | Claimed yes                             | No                                                  | No                                               |

*Three reconceptualizations of the same word — the arc this whole lecture has followed.*

# Timeline Summary

---

| Period | Key Figures               | Concept                     | Type                    |
|--------|---------------------------|-----------------------------|-------------------------|
| 1738   | Bernoulli                 | Utility as satisfaction     | Cardinal, psychological |
| 1870s  | Jevons, Menger, Walras    | Marginal utility            | Cardinal, psychological |
| 1906   | Pareto                    | Ordinal utility             | Ordinal, behavioral     |
| 1938   | Samuelson                 | Revealed preference         | Axiomatic (certainty)   |
| 1944   | Von Neumann & Morgenstern | Expected utility            | Axiomatic (risk)        |
| 1954   | Savage                    | Subjective expected utility | Axiomatic (uncertainty) |
| 1979   | Kahneman & Tversky        | Prospect theory             | Axiomatic (descriptive) |

# Where each field started

---

---

|                       | Economics                                                                                   | Psychology                                                                                                                          | Neuroscience                                                  |
|-----------------------|---------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------|
| Historical origin     | Bernoulli 1738 → Pareto 1906 → VNM 1944 — one continuous, increasingly mathematical lineage | Bentham 1789 (shared root) → four separate strands: Fechner 1860, Thorndike/Skinner 1898–1938, Tolman/Atkinson 1932–57, Maslow 1943 | 20th-c. reward-circuitry research — developed fully in Week 4 |
| Why study this at all | To predict/rationalize choice — a normative question from the start                         | Never one question: how behavior is learned, why motivation/risk-taking differs, what people need                                   | To locate and implement value in circuitry (Week 4)           |

# What each field says now

|                            | Economics                                                  | Psychology                                                                                                 | Neuroscience                                                |
|----------------------------|------------------------------------------------------------|------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------|
| What counts as “ worth”    | Utility, derived from consistent choice                    | No single fixed construct — reinforcement value, incentive value, need-satisfaction, subjective well-being | Reward, prediction-error signal (Week 4)                    |
| Assumption about stability | Preferences are fixed/constant                             | Preferences vary — and psychology says a lot about why                                                     | Largely open, tied to circuitry (Week 4)                    |
| Strength                   | Mathematically precise, predictive, testable               | Explains individual differences; the Atkinson 1957 parallel shows it independently found econ's shape      | Can potentially adjudicate between competing value theories |
| Weakness                   | Fixed-preference assumption — arguably econ's weakest link | No single agreed metric of “ worth” — a direct consequence of starting from four questions                 | Correlational more than explanatory, for now                |
| Ground truth?              | No                                                         | No                                                                                                         | No                                                          |

In 1957, a psychologist named Atkinson — working on why some people chase risk and others avoid it — wrote down:

$$\text{motivation} = \text{expectancy} \times \text{incentive value}$$

That is the exact same shape as expected utility — probability  $\times$  value — derived thirteen years after von Neumann and Morgenstern, apparently without either side reading the other.

*Two fields. Same shape. Completely different reasons for finding it.*

# Similar words, different meanings and histories

| Term            | Economics                               | Psychology                    | Neuroscience                   |
|-----------------|-----------------------------------------|-------------------------------|--------------------------------|
| Value / Utility | Bernoulli 1738 → Pareto 1906 → VNM 1944 | Fechner 1860 → Atkinson 1957  | Reward prediction error (Wk 4) |
| Gain            | Reference-dependence (Wks 7–8)          | Not this era's term (Wks 7–8) | —                              |
| Reward          | Not econ's vocabulary                   | Thorndike 1898 → Skinner 1938 | Dopamine / RPE (Wk 4)          |
| Wealth          | The default currency                    | Not used as a unit            | —                              |
| Need / Drive    | —                                       | Maslow 1943                   | —                              |
| Expectancy      | —                                       | Tolman 1932 → Atkinson 1957   | —                              |

# This week's additions

---

cardinal utility · diminishing marginal utility · ordinal utility · indifference curve · WARP · SARP · Von Neumann–Morgenstern axioms · representation theorem · certainty equivalent · risk premium · expectancy · need/drive

*Carried from Week 1: expected utility · loss aversion · framing effect · revealed preference · reward prediction error · warm-warm-glow utility · status signaling · risk · ambiguity · Knightian uncertainty · individual differences · risk aversion*