

Decision-Making Under Uncertainty

Week 2 — Value, Part 1: Behavioral and Economic Foundations

Lecturers:

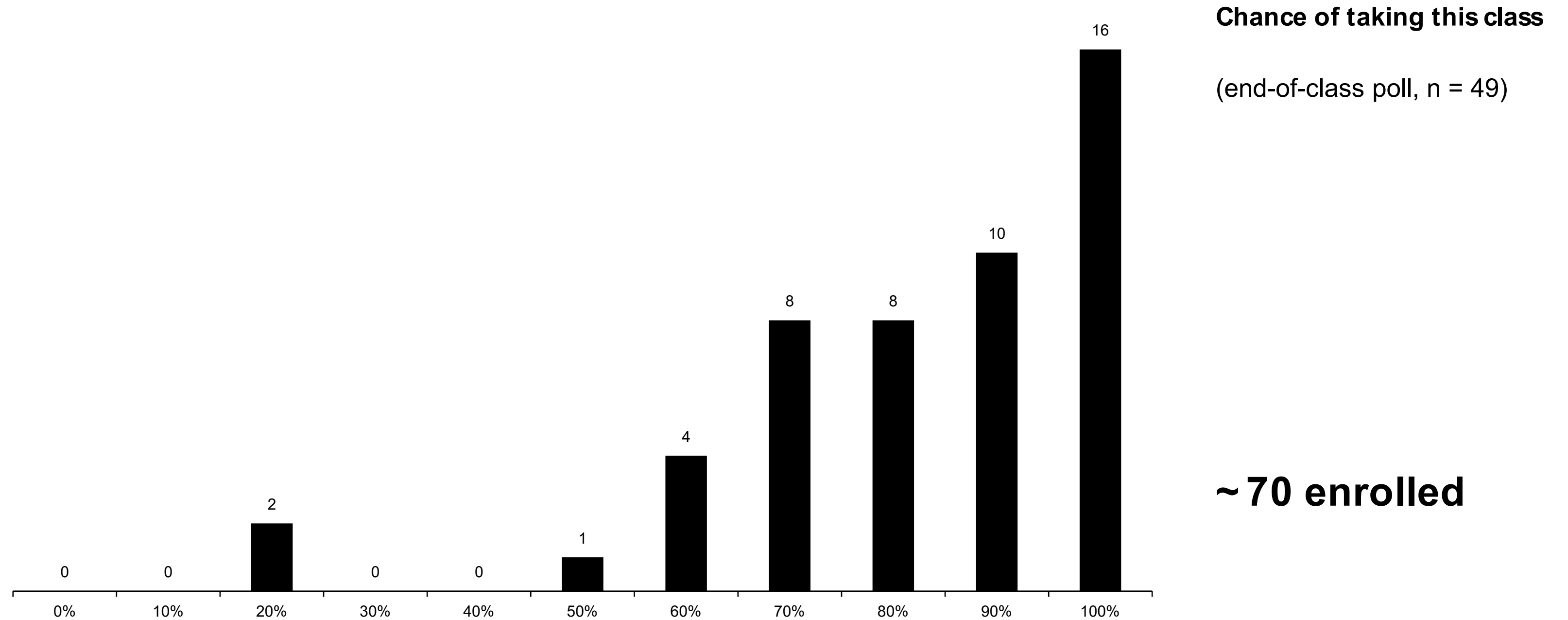
Prof. Kerstin Preuschoff

Dr. Marco Lehmann

TA: Jean-Paul Nohra

22.09.2026

What I value, and what I can count



What do you value?

In one word— no wrong answers, first thing that comes to mind.

 **Moodle -> Padlet — open text:** → post whatever you value, in your own words.

What Do You Value?

Use simple terms. ex: family, integrity, nature...

 ag
4 hours ago

family

 14

 Explorateur Toundra
4 hours ago

health

 13

 Explorateur Toundra
4 hours ago

time

 5

 om
4 hours ago

love



 om
4 hours ago

self-realization

 2

 Ospite Praterie
4 hours ago

time



 om
4 hours ago

self-fulfillment

 2

 Wolken Reiziger
4 hours ago

safety



 Icefall Climber
4 hours ago

fun



 AM
4 hours ago

discipline

 3

 Mon
4 hours ago

Ecology



 Courteous Anchovy
4 hours ago

friendship, emotional stability,
health, love,

 1

 Brilliant Beetle
4 hours ago

Happiness

 1

 IMD
4 hours ago

Football

 2

 Bâtitteur Pionnier
4 hours ago

effort

 1

 Hoogland Klauteraar
4 hours ago

friends

 2

 cc
4 hours ago

experience

 2

 Canvas Keeper
4 hours ago

family



 Gnetteur Bravoure
4 hours ago

Work



 Discovery Mapper
4 hours ago

Freedom

 4

Same three lenses, aimed at a decision

	Economics /Finance	Psychology	Neuroscience
Core question	Given constraints and information, what choice maximizes expected value/utility?	What cognitive and emotional processes produce this choice, and where does it depart from rational calculation?	What neural computations and circuits implement the choice?
What counts as “value”	Money, wealth, expected returns — one comparable currency.	Subjective well-being, satisfaction, felt importance — often domain-specific.	Reward and prediction-error signals — a candidate common neural currency.
Typical methods	Formal modeling, revealed preference, econometrics, natural experiments.	Controlled experiments, self-report, behavioral observation.	fMRI, EEG, single-unit recording, lesion studies.

No field has ground truth.

Similar words, different meanings and histories

Term	Economics	Psychology	Neuroscience
Value / Utility	Bernoulli 1738 → Pareto 1906 → VNM 1944	Fechner 1860 → Atkinson 1957	Reward prediction error (Wk 4)
Gain	Reference-dependence (Wks 7–8)	Not this era's term (Wks 7–8)	—
Reward	Not econ's vocabulary	Thorndike 1898 → Skinner 1938	Dopamine / RPE (Wk 4)
Wealth	The default currency	Not used as a unit	—
Need / Drive	—	Maslow 1943	—
Expectancy	—	Tolman 1932 → Atkinson 1957	—

Classical Economic Theories

"Father" of economics — and it's 'self-love,' not money

"It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard to their own self-interest. We address ourselves not to their humanity but to their self-love, and never talk to them of our own necessities, but of their advantages."

- **Adam Smith, The Wealth of Nations (1776), Book I, Chapter II**

From Blaise Pascal to Intuitive/Cardinal Utility (1600s-1870s)

Developing economic theories

- 1600's Blaise Pascal introduces probability theory by solving the following problem:
 - “Two people, A and B, agree to play a series of fair games until one person has won six games. They each have wagered the same amount of money, the intention being that the winner will be awarded the entire pot. But suppose, for whatever reason, the series is prematurely terminated, at which point A has won five games and B three. How should the stakes be divided?”
- This led to the development of probability theory and the concept of expected value.

Discussion: Knowing that probability and expected value had not yet been “invented” how would A and B have argued at the time? What solution would either one of them have proposed and why?

Computing expected value

$$EV(X) = \sum p(x) \cdot x$$

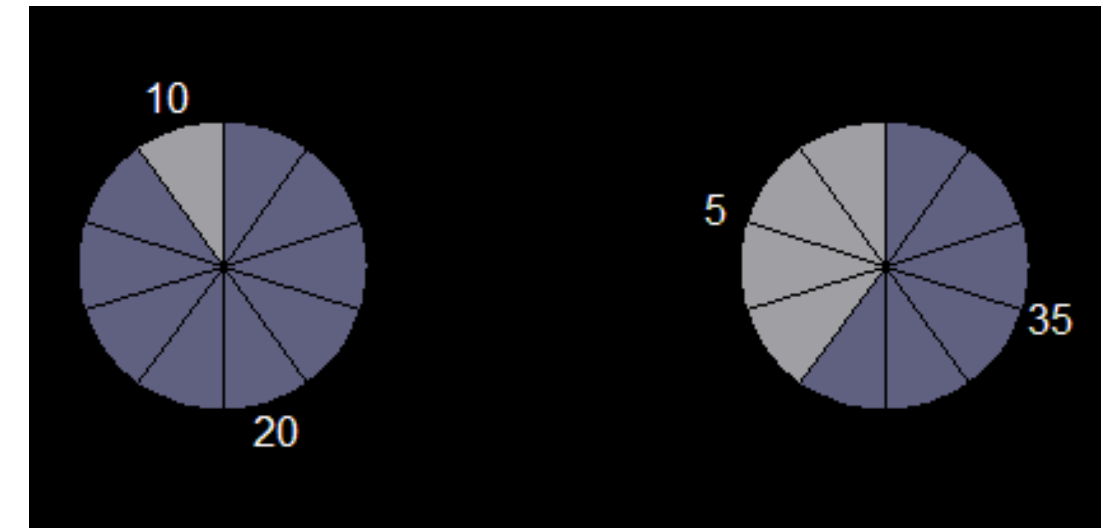
A probability-weighted average of possible outcomes.

Example 1: 10% chance of CHF 10, 90% chance of CHF 20

$$\rightarrow EV = 0.1 \times 10 + 0.9 \times 20 = \text{CHF } 19$$

Example 2: 40% chance of CHF 5, 60% chance of CHF 35

$$\rightarrow EV = 0.4 \times 5 + 0.6 \times 35 = \text{CHF } 23$$



What I value, and what I can count

Chance	# students	Probability	Contribution
0%	0	0.0	0.0
10%	0	0.1	0.0
20%	2	0.2	0.4
30%	0	0.3	0.0
40%	0	0.4	0.0
50%	1	0.5	0.5
60%	4	0.6	2.4
70%	8	0.7	5.6
80%	8	0.8	6.4
90%	10	0.9	9.0
100%	16	1.0	16.0

Total (n = 49):

**40.3 expected
attendees**

~70 students enrolled.

Today: p is your likelihood score, x is one more student in the room.

Falsifying expected value: The St. Petersburg Paradox

- First stated in 1713 by Nicolas Bernoulli, and solved in 1738 by his brother Daniel Bernoulli
- You are offered to buy a lottery ticket. The prizes in this lottery are determined by tossing a fair coin
- You receive:
 - CHF 2 if “heads” come up on the very first toss
 - CHF 4 if “tails” come up on the first toss and “heads” come up on the second toss
 - CHF 8 if “tails” come up on the first and the second toss and “heads” come up on the third toss
 - ... CHF 2^n if “heads” come up for the first time on the n th toss of the coin
- What is a maximum price that will you pay for this lottery ticket?

How would you compute the expected value (according to Pascal)?

Applying expected value theory to the problem

- Typically, people are willing to pay less than CHF 10 for the St. Petersburg lottery
- This lottery offers an outcome CHF 2^n with probability $1/(2^n)$, $n \in \mathbb{N}$
- The expected value of the St. P. lottery $EV(L) = 2 \times 1/2 + 4 \times 1/4 + \dots + 2^n \times 1/(2^n) + \dots = 1 + 1 + \dots + 1 = +\infty$
- **Someone who maximises expected value should offer all his wealth for the right to play in the St. Petersburg lottery**

Solving the problem (Bernoulli)

- Observe that the concept of expected value is not a descriptive decision theory
 - it fails to describe the observed behaviour as people do not bet all their wealth on the St. Petersburg lottery
- It is also not a normative decision theory: Should people ever bet all their wealth?
- A solution was proposed by Swiss mathematician Daniel Bernoulli in an article published in 1738 in the proceedings of St. Petersburg Academy of Sciences.

The dilemma with Pascal's expected value

- $EV(X) = \sum (p(x) * x)$
 - Option 1: CHF 50 for sure
 - Option 2: toss a coin, CHF 105 if heads, CHF 0 otherwise
- EV theory says that you should prefer option 2, many people prefer option 1

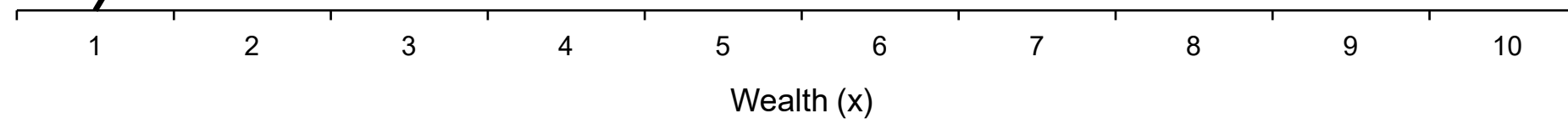
	Option 1	Option 2
Value	50	105
Probability	1	1/2
Expected value	50	52.5

Diminishing marginal utility

$$u(x) = \log(x)$$

The same increase in wealth adds less utility the more you already have.

“The second million doesn't feel like the first.”



Cf. Self-study. Weber-Fechner law

*“There is no doubt that a gain of one thousand ducats is
is more significant to a pauper than to a rich man, though
though both gain the same amount.”*

— Daniel Bernoulli, 1738

What kind of utility is this?

Bernoulli (1738)

Type	Cardinal
Meaning	Psychological — a real mental sensation
Measurable?	No — introspective only
Comparable across people?	Claimed yes

This table gains a column at §3 (Pareto) and again at §5 (von Neumann–Morgenstern) — full comparison recapped at close.

Why is water cheap and diamonds expensive?

"The things which have the greatest value in use have frequently little or no value in exchange; and, on the contrary, those which have the greatest value in exchange have frequently little or no value in use. Nothing is more useful than water: but it will purchase scarce any thing... A diamond, on the contrary, has scarce any value in use; but a very great quantity of other goods may frequently be had in exchange for it.

Adam Smith, The Wealth of Nations (1776)

	Total utility	M arginal utility	Price
Water	High	Low	Low
Diamonds	Low	High	High

Price tracks the value of the last unit consumed — not the value of the whole category.

➤ **Solved independently by 3 economists in the 1870s -> Jevons, Menger, Walras**

Marginal Utility Theory

- Marginal utility explains value
- Value = marginal utility (utility of the last unit consumed), not total utility
- Diminishing marginal utility - Each additional unit provides less satisfaction

Still Cardinal and Psychological

- Despite the advance, marginal utility theory still treated utility as:
 - Cardinal (measurable on an absolute scale)
 - Psychological (representing actual mental satisfaction)

The ordinal revolution: Vilfredo Pareto (1906)

Key Insight:

- Economics doesn't need cardinal utility!
- Only need to know **preference rankings**: $A > B > C$
- Don't need to know "how much" someone prefers A to B

Vilfredo Pareto's Argument:

- "All we observe is choice, not inner satisfaction"
- Economics should be based on observable behavior
- This is **ordinal utility** - only the ranking matters



?



The ordinal revolution: Vilfredo Pareto (1906)

Advantages:

- No need to measure "happiness"
- No interpersonal comparisons needed
- More scientifically respectable
- $utility(A)=10$, $utility(B)=9$, $utility(C)=8$ contains same information as $utility(A)=1000$, $utility(B)=50$, $utility(C)=1$

Transformation Property

- If $u(x)$ represents preferences, then $f(u(x))$ also represents the same preferences for any strictly increasing function f .

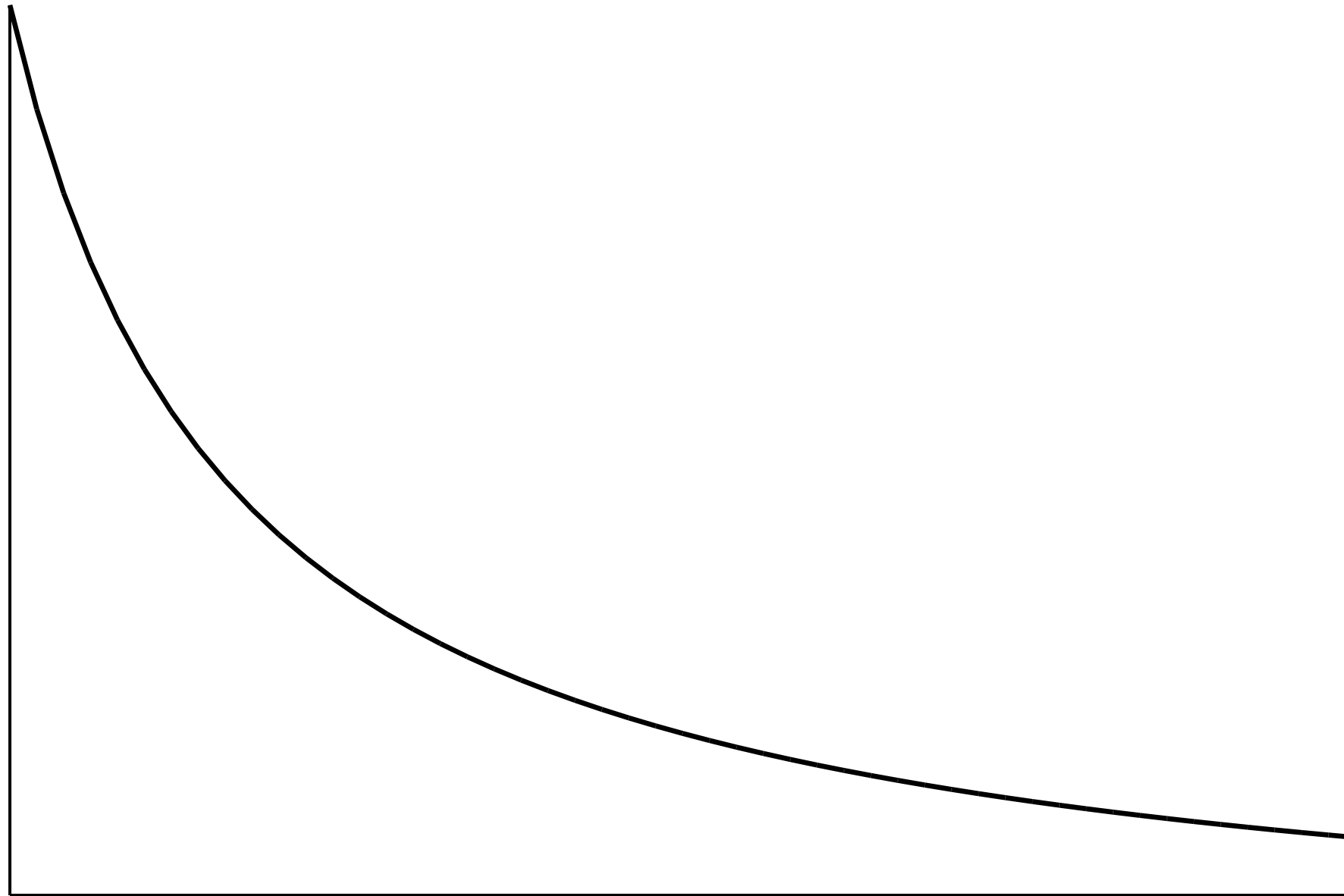


?



Ranking without measuring (illustrative)

↑ Perceived durability / values-fit



Every point on this curve is equally preferred.

You don't need a number for “ how someone values sustainability to describe their choice this way — only that they treat these bundles as interchangeable.

What kind of utility is this? (updated)

	Bernoulli (1738)	Pareto (1906)
Type	Cardinal	Ordinal
Meaning	Psychological — a real mental sensation	Mathematical convenience, not psychological reality
Measurable?	No — introspective only	Yes — revealed through choices
Comparable across people?	Claimed yes	No

Gains its final column at §5 (von Neumann–Morgenstern) — recapped in full at close.

The axiomatic revolution & Generalised axiomatisation

The Crisis of Utility Theory

Challenges

- How to handle choices under uncertainty? Ordinal utility works for certain outcomes, but what about risky choices?
- How to make utility theory rigorous? Need mathematical foundations
- How to avoid "mentalist" concepts? Unobservable mental states are problematic

Solutions

- Theory of Value (Hicks & Allen, 1934) -> finishes Pareto
- Revealed Preference Theory (Samuelson, 1938) -> removes theory itself
- Expected Utility Theory (von Neumann and Morgenstern, 1944) -> handles risk

The Crisis of Utility Theory. Solutions

Solutions

- Theory of Value (Hicks & Allen, 1934)
 - formalized Pareto insights which were purely conceptual; in practice economists were still using cardinal marginal utility
 - purify the *numbers* (cardinal $\rightarrow$ ordinal)
 - Moved from marginal utility to marginal rate of substitution
- Revealed Preference Theory (Samuelson, 1938)
 - purify the *existence claim* (no assumed preferences)
 - Just observe choice, everything can be extracted from this
- Expected Utility Theory (von Neumann and Morgenstern, 1944)
 - handles risk (NOT uncertainty in general)

Samuelson

- Start from simple, “primitive”, assumptions on preferences.
- Use observed choices, NOT UTILITY
- Derive preference ordering from choice consistency.
- Construct mathematical structure of choice.

Key Achievement:

- No mention of utility, satisfaction, or mental states
- Pure behaviorism
- If choices satisfy WARP, we can say they're "consistent with utility maximization"
- "As if" methodology - acts as if maximizing some utility function
- Normative models that focus on idealised choices and efficient allocation of resources.
- (as opposed to necessarily seeking to describe how people choose, as psychologists do, and how markets work).

Describing choice



?



Representing choice



10

>



9

>

The Axiomatic Approach to Modeling

Goal: Formalize models through representation theorems

i.e., find mathematical structures that “represent” the choices we observe

e.g.

$$U(x) > U(y) \iff x \succ y$$

numbers

preferences

articulate a set of necessary & sufficient conditions for representation

e.g., “transitivity,”

$$a \succ b, b \succ c \Rightarrow a \succ c$$

Representing choice



X

?



?

9

Representing choice



8

∧



9

∧

Inferring choice



>



<



8

>

<



10

Observing and inferring from choice



>



<



>



utility(bananas) = 8 < utility(apples) = 10

Utility functions

- used to represent individual's choice
- any function that results in the correct ordering works
e.g., $\text{utility}(\text{banana})=13$, $\text{utility}(\text{orange})=70$, $\text{utility}(\text{apple}) = 1053$
- only represent choice and have no independent explanatory power
i.e., I cannot extract the value of an option from its utility
- in their “purest” form they are merely a tool to capture choice
- different from *expected utility* discussed above

Consistency axioms

- GARP* - an individual's choices are transitive
- WARP* - if an individual chooses an orange over an apple when both are available in situation 1, then the individual should not choose the apple over the orange in another situation where again both are available
- questions about the neural and psychological processes behind choices are irrelevant as long as human decision making obeys such simple consistency axioms

*generalized/weak axiom of revealed preference
Houthakker, 1950

The beauty and the irony of revealed preference

- The beauty:
based on very few, very simple axioms, economists can make predictions about surprisingly complex choice patterns
predicts what new choices could possibly follow from an observed set of previous choices
- The irony:
it represented the “nail in the coffin” for psychological influences on economics

Von Neumann & Morgenstern (1944)
"Theory of Games and Economic Behavior"

The problem

- Ordinal utility works for choices under certainty
- But under **uncertainty**, we need more structure
- Example: If I prefer A to B to C, how do I choose between:
 - Lottery 1: 50% chance of A, 50% chance of C
 - Lottery 2: 100% chance of B?

Which would you choose?

Pair 1

A: CHF 3,000 for sure

B: 50% chance of CHF 7,000, 50% chance of CHF 0

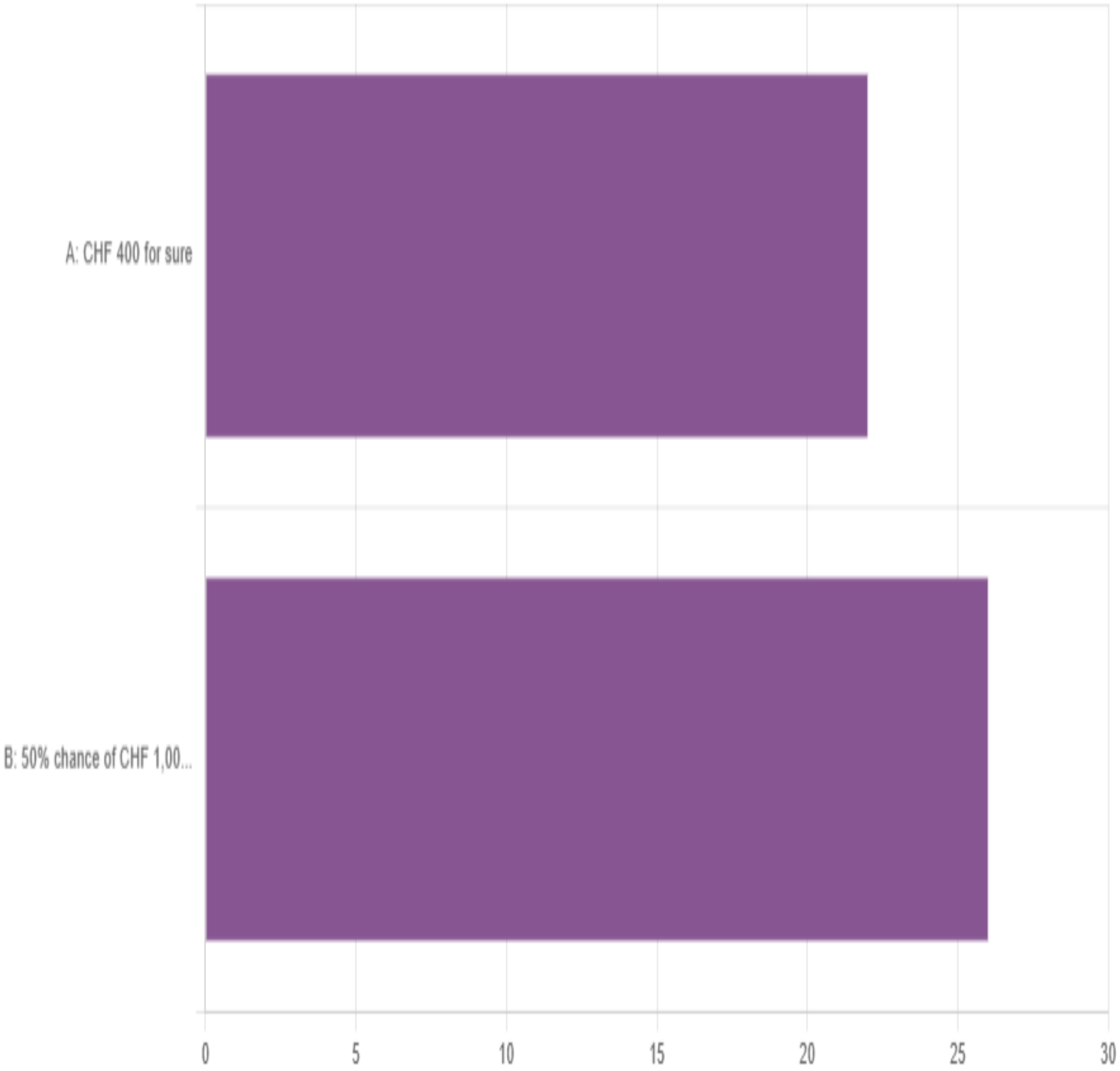
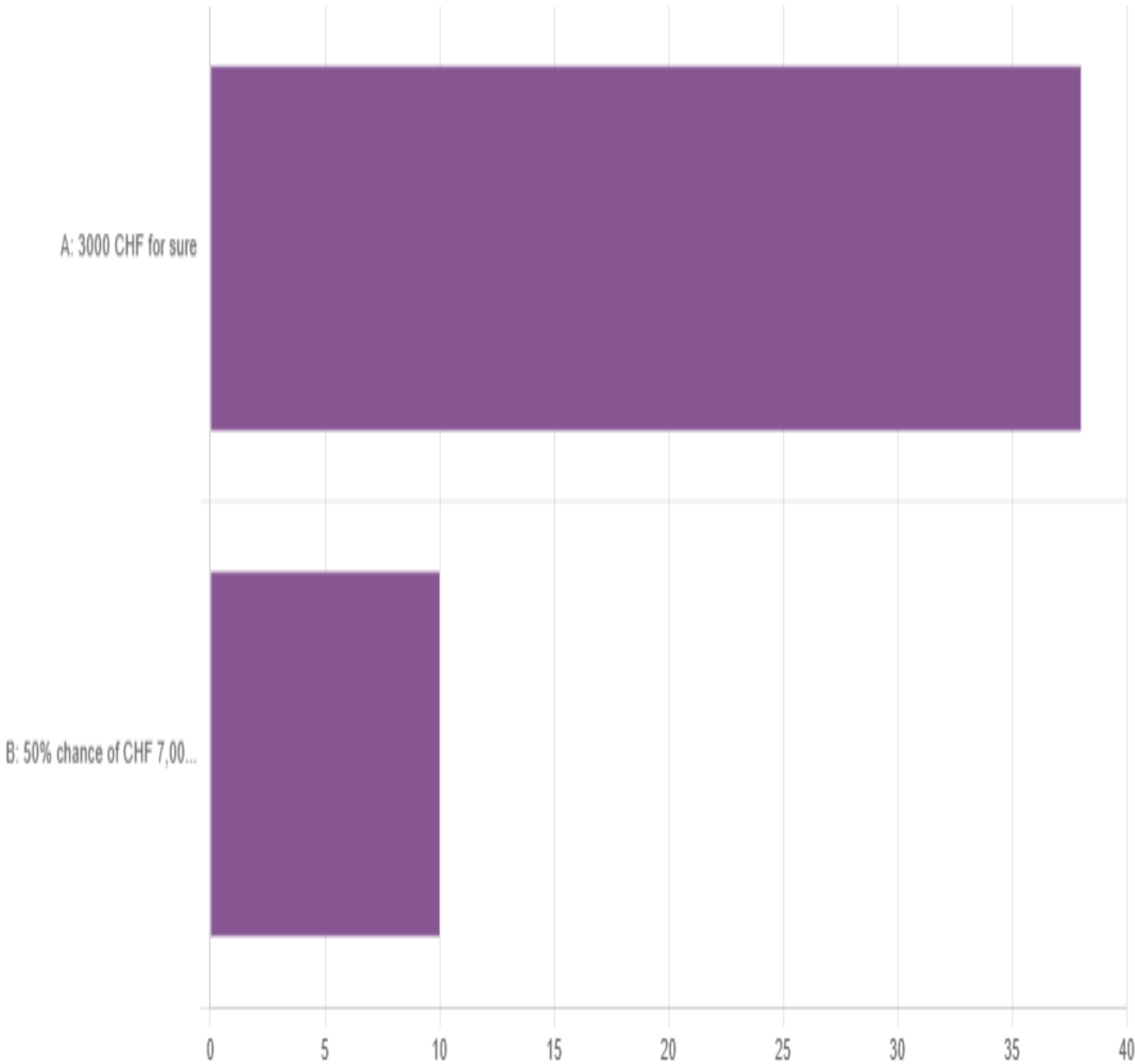
Pair 2

A: CHF 400 for sure

B: 50% chance of CHF 1,000, 50% chance of CHF 0

 Moodle — for each pair: → A or B?

Which would you choose?



The solution: The axiomatic approach

They proposed **axioms** for rational choice under uncertainty:

- 1. Completeness:** Can compare any two lotteries
 - 2. Transitivity:** If $A \succcurlyeq B$ and $B \succcurlyeq C$, then $A \succcurlyeq C$
 - 3. Continuity:** If $A \succ B \succ C$, there exists probability p such that $B \sim pA + (1-p)C$
 - 4. Independence:** If $A \succ B$, then $pA + (1-p)C \succ pB + (1-p)C$ for any lottery C and probability p (*note: see Allais paradox, Week 7*)
- Also showed how to recover utility from choices

The representation theorem

If preferences satisfy completeness, transitivity, continuity, and independence, exists a utility function $u(\cdot)$ such that:

$$U(L) = \sum p(x) \cdot u(x)$$

and the decision-maker acts *as if* maximizing expected utility.

$u(\cdot)$ is unique only up to a positive affine transformation: if $u(x)$ represents these preferences, so does $a \cdot u(x) + b$ for any $a > 0$.

Recovering utility from choices

Observed choice	What it reveals
Prefers CHF 3,000 for sure over 50/50 of CHF 7,000 or CHF 0	$u(3000) > 0.5 \cdot u(7000)$
Indifferent between CHF 400 for sure and 50/50 of CHF 1,000 or CHF 0	$u(400) = 0.5 \cdot u(1000)$

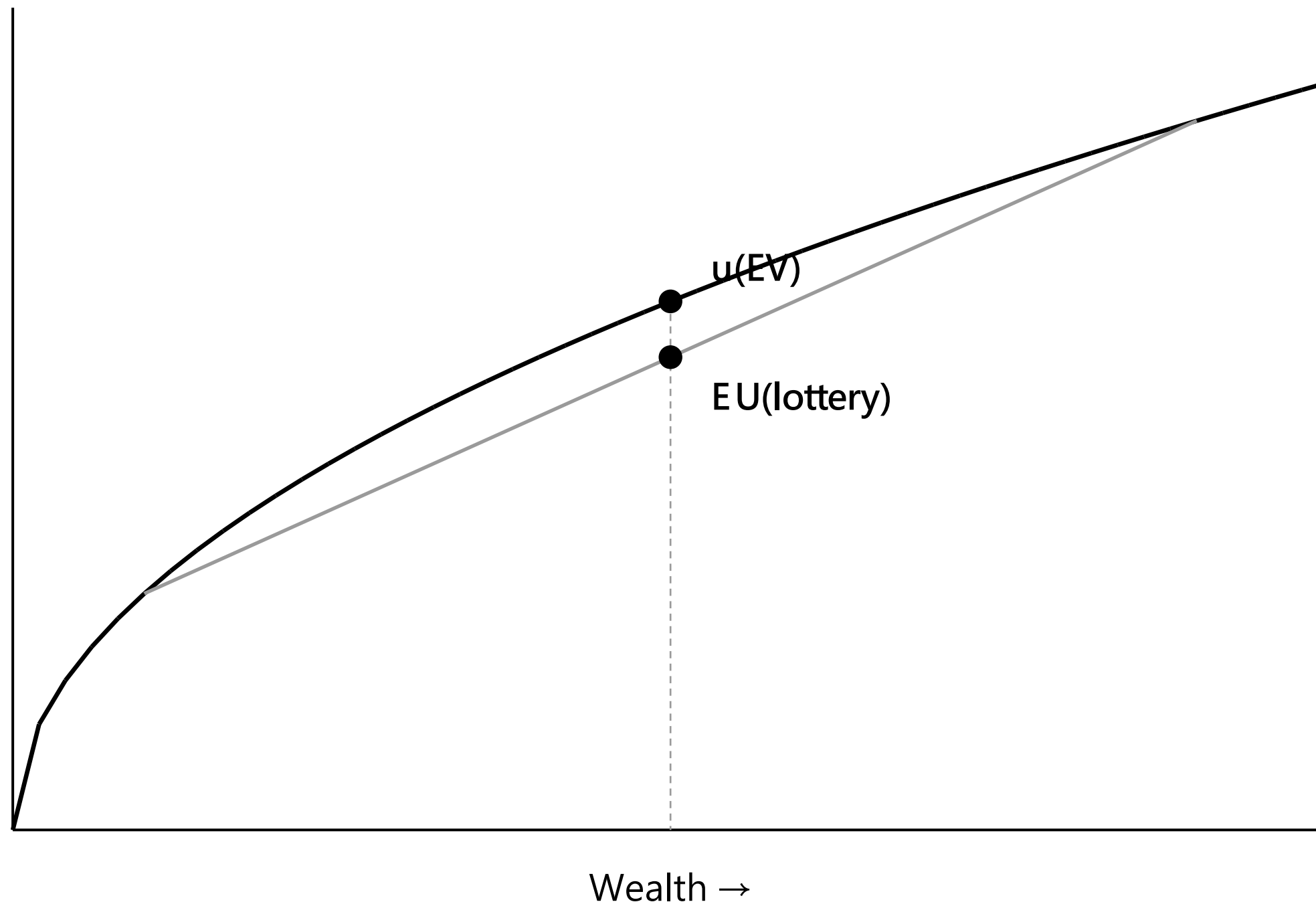
Normalize $u(0) = 0$, $u(7000) = 1$:

→ $u(3000) > 0.5$ — that inequality *is* risk aversion, numerically.

→ $u(400) = 0.5$

NOTE: two separate rulers (one per gamble), not one scale.

Concavity is risk aversion



A concave $u(\cdot)$ means:

$$EU(\text{lottery}) < u(\text{EV of lottery})$$

The room's own preference for the certain amount over the higher-EV gamble *is* this curve, visually.

What kind of utility is this? (final)

	Bernoulli (1738)	Pareto (1906)	Von Neumann–Morgenstern (1944)
Type	Cardinal	Ordinal	Cardinal — again, different meaning
Meaning	Psychological — a real mental sensation	Mathematical convenience, not psychological reality	Derived from consistent preferences, not assumed
Measurable?	No — introspective only	Yes — revealed through choices	Yes — via lottery choices
Comparable across people?	Claimed yes	No	No

Three reconceptualizations of the same word — the arc this whole lecture has followed.

Timeline Summary

Period	Key Figures	Concept	Type
1738	Bernoulli	Utility as satisfaction	Cardinal, psychological
1870s	Jevons, Menger, Walras	Marginal utility	Cardinal, psychological
1906	Pareto	Ordinal utility	Ordinal, behavioral
1938	Samuelson	Revealed preference	Axiomatic (certainty)
1944	Von Neumann & Morgenstern	Expected utility	Axiomatic (risk)
1954	Savage	Subjective expected utility	Axiomatic (uncertainty)
1979	Kahneman & Tversky	Prospect theory	Axiomatic (descriptive)

Where each field started

	Economics	Psychology	Neuroscience
Historical origin	Bernoulli 1738 → Pareto 1906 → VNM 1944 — one continuous, increasingly mathematical lineage	Bentham 1789 (shared root) → four separate strands: Fechner 1860, Thorndike/Skinner 1898–1938, Tolman/Atkinson 1932–57, Maslow 1943	20th-c. reward-circuitry research — developed fully in Week 4
Why study this at all	To predict/rationalize choice — a normative question from the start	Never one question: how behavior is learned, why motivation/risk-taking differs, what people need	To locate and implement value in circuitry (Week 4)

What each field says now

	Economics	Psychology	Neuroscience
What counts as “ worth”	Utility, derived from consistent choice	No single fixed construct — reinforcement value, incentive value, need-satisfaction, subjective well-being	Reward, prediction-error signal (Week 4)
Assumption about stability	Preferences are fixed/constant	Preferences vary — and psychology says a lot about why	Largely open, tied to circuitry (Week 4)
Strength	Mathematically precise, predictive, testable	Explains individual differences; the Atkinson 1957 parallel shows it independently found econ's shape	Can potentially adjudicate between competing value theories
Weakness	Fixed-preference assumption — arguably econ's weakest link	No single agreed metric of “ worth” — a direct consequence of starting from four questions	Correlational more than explanatory, for now
Ground truth?	No	No	No

In 1957, a psychologist named Atkinson — working on why some people chase risk and others avoid it — wrote down:

$$\text{motivation} = \text{expectancy} \times \text{incentive value}$$

That is the exact same shape as expected utility — probability $\times$ value — derived thirteen years after von Neumann and Morgenstern, apparently without either side reading the other.

Two fields. Same shape. Completely different reasons for finding it.

Similar words, different meanings and histories

Term	Economics	Psychology	Neuroscience
Value / Utility	Bernoulli 1738 → Pareto 1906 → VNM 1944	Fechner 1860 → Atkinson 1957	Reward prediction error (Wk 4)
Gain	Reference-dependence (Wks 7–8)	Not this era's term (Wks 7–8)	—
Reward	Not econ's vocabulary	Thorndike 1898 → Skinner 1938	Dopamine / RPE (Wk 4)
Wealth	The default currency	Not used as a unit	—
Need / Drive	—	Maslow 1943	—
Expectancy	—	Tolman 1932 → Atkinson 1957	—

This week's additions

cardinal utility · diminishing marginal utility · ordinal utility · indifference curve · WARP · GARP · Von Neumann–Morgenstern axioms · representation theorem · certainty equivalent · risk premium · expectancy · need/drive

Carried from Week 1: expected utility · loss aversion · framing effect · revealed preference · reward prediction error · warm-warm-glow utility · status signaling · risk · ambiguity · Knightian uncertainty · individual differences · risk aversion